ESSAM SCHOOL IN APPLIED MATHEMATICS

ESSAM SCHOOL ON MATHEMATICAL ASPECTS OF FLUID FLOWS

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PLENARY SPEAKERS

Dominic Breit (*Heriot-Watt University Edinburgh*)
Yann Brenier (*Ecole Polytechnique, Palaiseau*)
Pierre-Emmanuel Jabin (*University of Maryland*)
Christian Rohde (*Universität Stuttgart*)

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Stochastic Navier-Stokes Equations

The dynamics of liquids and gases can be modeled by the Navier-Stokes system of partial differential equations describing the balance of mass and momentum in the fluid flow. In recent years their has been an increasing interest in random influences on the fluid motion modeled via stochastic partial differential equations.

In this course we will study the existence of weak martingale solutions to the stochastic Navier-Stokes equations (both incompressible and compressible). These solutions are weak in the analytical sense (derivatives exists only in the sense of distributions) and weak in the stochastic sense (the underlying probability space is not a priori given but part of the problem).

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Concepts of Generalized Solutions in Incompressible Fluid Mechanics

Lecture I: Measure-valued and dissipative solutions.
The concepts of measure-valued solutions and dissipative solutions were respectively introduced in the late 80s early 90s by DiPerna-Majda and Lions for the Euler equations of incompressible fluids. Although they look very different from each other, I will explain their close relationship in relation with the "weak-strong" uniqueness principle.

Lectures II and III: Variational and sharp measure-valued solutions to the Euler equations.
Using the least action principle, one may try to solve the Euler equation not as a Cauchy problem but rather as a minimization problem. This opens the way to a rather precise "sharp" concept of measure-valued solutions. Its relevence for the Cauchy problem will be discussed.

Lecture IV: Magnetic relaxation of the Euler equations and dissipative solutions.
Following K. Moffatt one can look for stationary solutions to the Euler equations with prescribed topology by solving a degenerate parabolic equations coming from MHD theory. This leads to a suitable concept of generalized solutions blending variational features with Lions' concept of dissipative solutions.

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Quantitative Regularity Estimates for Compressible Fluids

The aim of this course is to present some of the classical and more recent techniques to control oscillations and measure the regularity of weak solutions to various models of compressible fluids and in particular: The barotropic compressible Navier-Stokes and some Navier-Stokes--Fourier systems.

We will start by introducing the regularity theory on the simpler case of linear advection equations which follows from the notion of renormalized solutions. We will then review how the estimates developed for compressible fluids, first by P.L. Lions and then by E. Feireisl for the compressible Navier-Stokes, and E. Feireisl and A. Novotny for the Navier-Stokes-Fourier system. We will finish by presenting some recent estimates introduced with D. Bresch.

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Diffuse Interface Modelling for Two-Phase Flow

Phase field or diffuse interface approaches are frequently used to model the complex dynamics of two-phase flow problems. The major advantage of this ansatz is the ability to describe critical phenomena like topological changes due to merging/splitting of single phases or the phases' interaction with walls. In the last years there have been substantial advancements in the theory and numerics for diffuse-interface models. The most important directions for compressible free flows will be discussed concerning well-posedness and asymptotic behaviour for various limit regimes including the classical sharp interface limit. Besides the purely analytical issues it is important that diffuse interface models allow a reliable and efficient numerical approximation. This imposes new constraints on the whole model ansatz. While there is by now a quite well-established theory for compressible free flow situations the theory for two-phase flows in porous media is still in its very beginnings. Basic new developments for least partially homogenized flow of multiple phases and components are presented.